Kasetsart J. (Nat. Sci.) 38 : 409 - 418 (2004)

Development of One Dimensional Implicit Dynamic Wave Model

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ABSTRACT

This study had an objective to develop a one dimensional implicit dynamic wave model. The model was named a dynamic wave model (DYMWAV). Visual Basic.Net program was used to develop the model programming. The DYMWAV model was applied to the Bang Pakong River Basin to investigate the model performance for the flow situations affected by backwater and tide. The results showed that the DYMWAV model simulated these flow situations satisfactorily. The performance of the DYMWAV model was then compared with the performance of the Hydrodynamic Model, a module of the MIKE 11 model (MIKE 11 HD), which was generally accepted for unsteady flow simulation. This was carried out by applying the MIKE 11 HD model on the identical case study as applied to the DYMWAV model. The Manning's n for each cross-section and each river reach calibrated from the DYMWAV model was applied to the MIKE 11 HD model. The simulation results from both models showed well corresponding flow hydrographs with little discrepancies.

Key words: Saint-Venant equations, dynamic wave, implicit finite difference, nonlinear algebraic equations, backwater effect, tidal effect, Bang Pakong River Basin

INTRODUCTION

From the past till now, the researchers have developed mathematical models to analyze one dimensional channel routing in many forms. The basic equations are the Saint-Venant equations which are nonlinear partial differential equations. Because of the complicated nonlinear equations solution, some researchers have tried to reduce the complication by changing nonlinear equations to linear equations for simpler methods of solution. For instant, Meijer *et al*. (1965) developed a mathematical model called "Node and Branch" to simulate a channel network system. This model was applied to the channels connected to two large reservoirs. Torranin (1969) developed a one dimensional mathematical model for solving steady flow problems in an open channel which was a single reach. Chatrcharoenmitr (1977) developed a mathematical model to simulate flow in the Chao Praya River between the Chao Praya Dam and Bang Sai by combining a storage flood plain model and the Node and Branch model with the weir equation. Vatcharasinthu (1977) developed a one dimensional mathematical model for a single reach to explain flow in channel and flood plain. Tingsanchali and Arbhabhirama (1978) developed a model for a single reach by mixing the model developed by Torranin (1969) and the Node and Branch with storage flood plain model developed by Chatrcharoenmitr (1977). This model could simulate flow situations of channel system as a network and was used for estimating flood and studying water level affected by river dredging.

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Received date : 13/05/04 Accepted date : 30/06/04

This research developed a one dimensional implicit dynamic wave model to simulate one dimensional unsteady flow situation with backwater and tidal effects by using the same equations (the Saint-Venant equations). In contrast, instead of changing the nonlinear equations to the linear equations as applied for the models mentioned above, the nonlinear problems was directly solved by using an implicit weighted four-point finite difference approximation with the Newton-Raphson method, which was an iteration technique for solving nonlinear equations.

MATERIALS AND METHODS

Basic equations

The basic equations of a one dimensional dynamic wave model are the Saint-Venant equations describing one-dimensional unsteady open-channel flow. The equations consist of two equations, i.e. the continuity equation and the momentum equation, which can be mathematically expressed as;

Continuity equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \tag{1}$$

Momentum equation

$$\frac{\partial Q}{\partial t} + \frac{\partial\left(\beta Q^2 / A\right)}{\partial x} + gA\left(\frac{\partial h}{\partial x} + S_f + S_e\right) - \beta q v_x + W_f B = 0 \tag{2}$$

where x is longitudinal distance along the channel, t is time, Q is discharge, A is cross-sectional area of flow, q is lateral inflow per unit length of the channel, h is water surface elevation, v_x is velocity of lateral flow in the direction of channel flow, S_f is friction slope, S_e is eddy loss slope, B is channel width, W_f is wind shear force, β is momentum correction factor, and g is gravitational acceleration.

Solution methods

The Saint-Venant equations are nonlinear partial differential equations expressed in both spatial and time derivatives as shown in equations (1) and (2). These equations are the hyperbolic partial differential equations that cannot be solved using the analytical solution, but can be solved by numerical schemes. In this research, the implicit weighted four points finite difference approximation was used to solve the equations. In the equations solving processes, the adjoining two cross-sections can be organised to change the system of partial differential equations into the system of finite difference equations as shown in equations (3) and (4).

Continuity equation

$$\begin{aligned}&\theta\left(Q_{i+1}^{j+1} - Q_i^{j+1} - \bar{q}_i^{\,j+1}\Delta x_i\right) + (1-\theta)\\ &\left(Q_{i+1}^{j} - Q_i^{j} - \bar{q}_i^{\,j}\Delta x_i\right) + \frac{\Delta x_i}{2\Delta t_j}\\ &\Big[\left(A + A_o\right)_i^{j+1} + \left(A + A_o\right)_{i+1}^{j+1}\\ &-\left(A + A_o\right)_i^{j} - \left(A + A_o\right)_{i+1}^{j}\Big] = 0\end{aligned} \tag{3}$$

Momentum equation

$$\begin{aligned}&\frac{\Delta x_i}{2\Delta t_j}\left(Q_i^{j+1} + Q_{i+1}^{j+1} - Q_i^{j} - Q_{i+1}^{j}\right)\\ &+\theta\left\{\left(\frac{\beta Q^2}{A}\right)_{i+1}^{j+1} - \left(\frac{\beta Q^2}{A}\right)_{i}^{j+1} + g\bar{A}_i^{\,j+1}\right.\\ &\left[h_{i+1}^{j+1} - h_i^{j+1} + \left(\bar{S}_f\right)_i^{j+1}\Delta x_i + \left(\bar{S}_e\right)_i^{j+1}\Delta x_i\right]\\ &\left.-\left(\overline{\beta q v_x}\right)_i^{j+1}\Delta x_i + \left(\overline{W_f}\,\bar{B}\right)_i^{j+1}\Delta x_i\right\}\\ &+(1-\theta)\left\{\left(\frac{\beta Q^2}{A}\right)_{i+1}^{j} - \left(\frac{\beta Q^2}{A}\right)_{i}^{j}\right.\\ &+g\bar{A}_i^{\,j}\left[h_{i+1}^{j} - h_i^{j} + \left(\bar{S}_f\right)_i^{j}\Delta x_i + \left(\bar{S}_e\right)_i^{j}\Delta x_i\right]\\ &\left.-\left(\overline{\beta q v_x}\right)_i^{j}\Delta x_i + \left(\overline{W_f}\,\bar{B}\right)_i^{j}\Delta x_i\right\} = 0\end{aligned} \tag{4}$$

where i is the cross-section index, for 1,2,…,N, numbering from upstream to downstream, j is the present time and $j+1$ is the advance time.

Because of the nonlinearity nature of the equations, the solutions of equations (3) and (4) are obtained by using an iterative procedure. The Newton-Raphson iterative method for solving the nonlinear system is used.

Development of a one dimensional implicit dynamic wave model

Development of a one dimensional implicit dynamic wave model is programed by the Visual Basic.Net program. The developed model is named DYMWAV (Dynamic Wave) Version 1.0. Details of model processing can be summarized as the following.

1. Data input

Data input for the model can be classified into 3 groups as the followings:

1.1 Model parameters consisting of a distance interval (Δx), a time interval (Δt) and an implicit coefficient (θ)

1.2 Time series of the hydrological and hydrodynamic data assigned as the boundary and initial conditions

1.3 Cross-section and roughness coefficients (Manning's n) for the main channel and its tributaries.

2. Data analysis

Data analysis was separated as the following.

2.1 Cross-sectional analysis

The analysis of cross-section characteristics comprises the channel width (B), cross-sectional area (A), hydraulic radius (R), and wetted perimeter (P).

2.2 Time series analysis

Time series data such as hydrograph were calculated using interpolation technique for each time interval for the main channel and its tributaries.

2.3 Initial condition analysis

Initial values for the variables at time line j were evaluated. These comprised the values of Q_i^j, Q_{i+1}^j, h_i^j, h_{i+1}^j, A_i^j, A_{i+1}^j, B_i^j and B_{i+1}^j in equations (3) and (4) for the main channel and its tributaries.

3. Channel network determination

This routine was to determine whether a channel network was a single channel or a channel with tributaries. In case of a channel with tributaries, it would be assumed the tributaries flow as lateral inflow (q^{**}) of the main channel before analyzing the main channel discharge and water level. If it is a single channel, it would go directly to the next step.

4. Discharge and water level analysis

The Newton-Raphson method is used for discharge and water level analysis. Equation system for an iteration of the Newton-Raphson method for a river is formulated as expressed in equation (5) and Figure 1.

Equation system for an iteration of the Newton-Raphson method for a river with 4 reaches (Chow *et al.*, 1988)

$$[Z][\Delta_i] = -[F_i] \qquad (5)$$

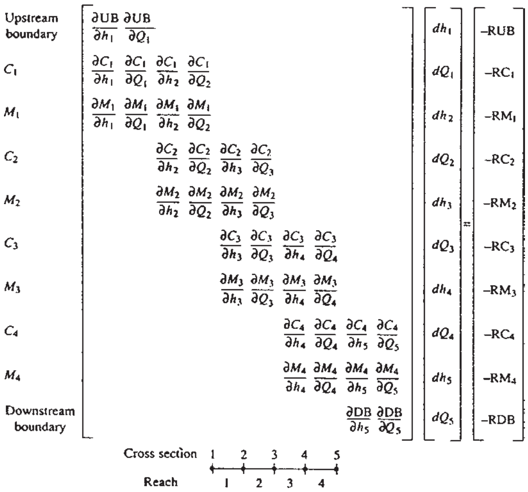

Figure 1 Equation system for an iteration of the Newton-Raphson method for a river with 4 reaches (Chow *et al.*, 1988).

where [Z] represents the Jacobian coefficient matrix, Δ_i are flow variables, i.e. dQ and dh, and [F_i] represents vector of the negative residuals.

The steps of model computation are described as the following.

4.1 The beginning unknown terms are assumed at time line *j+1*. The unknown terms in equations (3) and (4) are Q_i^{j+1}, Q_{i+1}^{j+1}, h_i^{j+1}, h_{i+1}^{j+1}, A_i^{j+1}, A_{i+1}^{j+1}, B_i^{j+1} and B_{i+1}^{j+1}, where A and B are not flow variables but can be expressed as the functions of flow variables Q_i^{j+1}, Q_{i+1}^{j+1}, h_i^{j+1} and h_{i+1}^{j+1} described in the function $x^k = \left(Q_i^k, h_i^k, \ldots, Q_N^k, h_N^k\right)$, where k is time line and x is a non-flow variable.

4.2 The residual terms are computed as the negative value of the residuals (RUB^k, $RC_1{}^k$, $RM_1{}^k$, …, $RC_{N-1}{}^k$, $RM_{N-1}{}^k$ and RDB^k) by substituting any known variables such as initial values into equations (3) and (4).

4.3 All terms in the Jacobian coefficient matrix are derived from the derivatives of the continuity and momentum equations and at the upstream and downstream boundary conditions, as shown in Figure 1.

4.4 The matrix equation (equation (5)) is solved by using the Gaussian elimination method for dh_i and dQ_i.

4.5 The next time line values of the water level (h_i^{k+1}) and discharge (Q_i^{k+1}) are calculated at various points along the channel by using equations (6) and (7) as shown:

$$h_i^{k+1} = h_i^k + dh_i \qquad (6)$$

$$Q_i^{k+1} = Q_i^k + dQ_i \qquad (7)$$

4.6 The convergent condition is checked by using the following criterion;

$$\left| x^{k+1,m+1} - x^{k+1,m} \right| < \in$$

where m is an iterative number, 1,2,3, …, and $\in$ is a tolerance value.

5. The discharge and water level analysis is performed along the main channel and at the confluences between the main channel and its tributaries (if any).

6. In case of the river with tributaries, the model will consider each tributary as a main channel. The discharge and water level will be calculated at each tributary (*q*) by using computed water levels from step 5 at the confluence between the tributary and the main channel as a downstream boundary condition. Another convergent condition is established for this case by comparing the assumed lateral inflow and calculated lateral inflow using the following criterion;

$$\left| q - q^{**} \right| \le \in$$

If the criterion is not met, an iteration algorithm (Fread, 1973) will be performed by using new lateral inflow (*q**) as described in equation (8) and the calculation process will start again from step 3.

$$q^* = \alpha q + (1-\alpha) q^{**} \qquad (8)$$

where is a weighting factor which is between zero and one.

Model calibration and model performance

Model calibration has an objective to find an appropriate Manning's n of each cross section that represents natural behaviors of the river flows. The calibration results of both discharges and water levels from the DYMWAV model will be compared with observational discharges and water levels at various positions along the river. The comparison results between both data have to be within an acceptable level. However, if the differences are too large, the Manning's n has to be adjusted until the differences are within the acceptable limit.

To test the performance of the DYMWAV model for the river flow simulation, the MIKE 11 HD model was applied for the same study area with the same data and the same Manning's n calibrated using the DYMWAV model. If the differences between discharge or water level calculated by both models are statistically insignificant and within an acceptable limit, it can be concluded that the DYMWAV model can be applied for other study areas.

Study area and input data

In this research, the study area is the Bang Pakong River Basin where the flow situation has backwater and tidal effects. The Bang Pakong River consists of two main tributaries that are the Prachin Buri River and the Nakhon Nayok River. The Bang Pakong River Basin has a catchment area of approximately 8,679 km^2 and the total channel length is approximately 115 km. The boundary of the study river begins from the gauging station Kgt.3 as an upstream boundary control to an estuary of the Bang Pakong river as a downstream boundary control. The schematic diagram of the Bang Pakong River and its tributaries is shown in Figure 2. Flow hydrographs of the upstream boundaries and hourly water levels at the downstream boundary and along the Bang Pakong river are used for model simulation. Flood event that considered in this study is between July 9, 1997 and October 27, 1997. Lateral inflows for various sub-basins along the Bang Pakong River and its tributaries, without gauging stations, were estimated using the unit hydrograph technique.

Criteria for assessing model performance

Model performance can be assessed by a comparison between the simulated water levels and the observed water levels at different gauging stations. In this study, two statistical measures are used. They are a correlation coefficient (r) and an efficiency index (EI) as shown in equations (9) and (10).

$$r = \frac{\Sigma\left(H_i - \overline{H}\right)\cdot\left(F_i - \overline{F}\right)}{\sqrt{\left[\Sigma\left(H_i - \overline{H}\right)^2 \cdot \Sigma\left(F_i - \overline{F}\right)^2\right]}} \tag{9}$$

$$EI = \frac{\sum_{i=1}^{N}\left(H_i - \overline{H}\right)^2 - \sum_{i=1}^{N}\left(H_i - \overline{F_i}\right)^2}{\Sigma\left(H_i - \overline{H}\right)^2} \times 100\% \tag{10}$$

where H_i is observed water level at time i, $\overline{H}$ is an average of observed water level, F_i is simulated water level at time i, $\overline{F}$ is an average of simulated water level, and N is the number of data.

RESULTS AND DISCUSSION

The results of DYMWAV model applied to the Bang Pakong River Basin

1. The results of DYMWAV model calibration

The DYMWAV model calibration of flow hydrograph event between July 9, 1997 and October 27, 1997 was performed by adjusting the Manning’s n values so that the simulated stage hydrographs were close to the observed data at each gauging station. The calibration results showed that the Manning’s n values of the Bang Pakong River were mainly 0.030, except along the river reach between the chainage 124.62 and 181.38 km where the Manning’n values were 0.020. For the Prachin Buri River between the chainage 0.00 and 66.69 km and between 66.70 and 124.62 km, the Manning’n values were 0.035 and 0.028, respectively. The model calibration results were generally satisfied. The correlation coefficients (r) were mainly between 0.702 and 0.957 and the efficiency index (EI) were mainly between 70.00 and 87.06. Except in some gauging stations, the statistical results were low, e.g. at the Tah Khai Regulator, the correlation coefficient (r) was 0.650, at the Ta Tour, EI was only 60.05. However, this research had main objective to develop the model which emphasize in the model performance checking rather than the model calibration. The reliability of the DYMWAV model performance was checked by comparing the model results with the MIKE 11 HD results on the same situation. Therefore the model calibration just used to estimate the Manningûs n of each cross section approximately for using in the model performance checking step. Although the statistical results of the model calibration in some gauging station were low, the result of the DYMWAV model

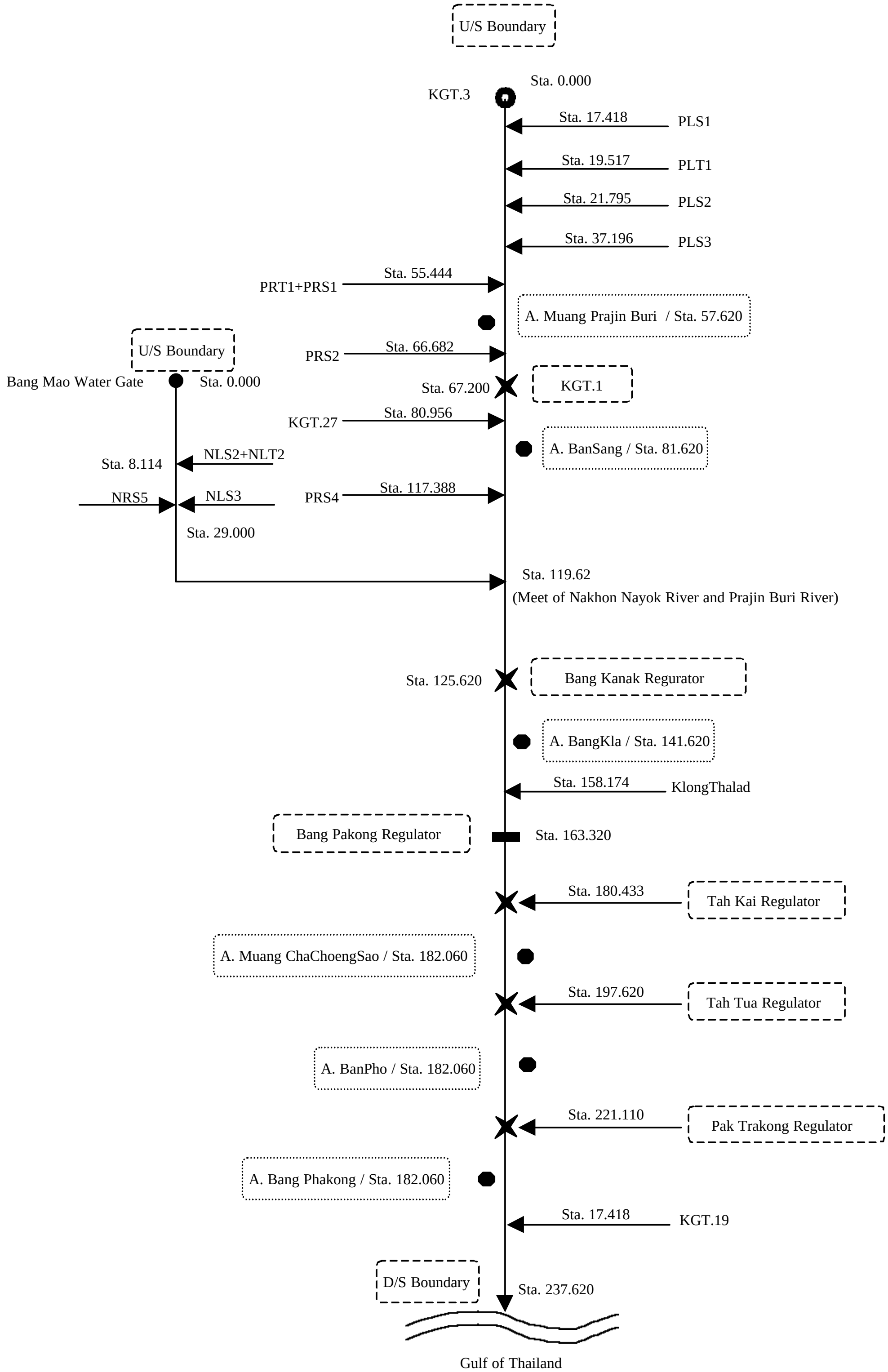

Figure 2 A schematic diagram of the Bang Pakong River and its tributaries.

calibration could be acceptable.

2. The results of the models performance comparison

The DYMWAV model performance was checked with the MIKE 11 HD model performance on the same case study of the Bang Pakong River. The Manning's n values calibrated using the DYMWAV model was applied in the MIKE 11 HD model. The simulation results from both models showed well corresponding stage hydrographs. The correlation coefficient values (r) were between 0.945 and 0.993, and the efficiency index values (EI) were between 85.75 and 97.30. The comparison results were shown in Figures 3 to 7. The statistical results of the model performance comparison of every gauging stations were high. Therefore the DYMWAV model performance was close to the MIKE 11 HD model performance and it can conclude that the DYMWAV model can simulate unsteady flow situations reasonably well.

CONCLUSION AND RECOMENDATION

The DYMWAV model is a one dimensional dynamic wave model developed for simulating one dimensional unsteady flow situation with backwater or tidal effects. The basic equations of the DYMWAV model are the Saint-Venant equations which are the nonlinear partial differential equations. These equations were solved by using the implicit weighted four points finite difference approximation. The Newton Raphsonís method (an iteration method) was used to solve the nonlinear equations. The DYMWAV model was applied to the Bang Pakong River Basin to investigate the model performance for the flow situations affected by backwater and tide. The results showed that the DYMWAV model simulated these flow situations satisfactorily with corresponding stage hydrographs between the observed stage hydrographs and the simulated stage hydrographs. The statistical results showed that in general, the DYMWAV model can simulate unsteady flow situations reasonably well. The reliability of the DYMWAV model performance was assessed by comparison with the MIKE 11 HD model performance. The comparison results showed that the DYMWAV model results were close to those of the MIKE 11 HD model. Therefore, the DYMWAV model developed under this research can simulate one dimensional unsteady flow situation and can be used to replace a commercialize software like the MIKE 11 HD model which has high cost.

Even though the DYMWAV model can simulate one dimensional unsteady flow situations close to the ability of the MIKE 11 HD model, the developed model performance and its facilities should still be improved. Firstly, since the model cannot be used to simulate flow situations with hydraulic structures situated along the channels, therefore this facility should be added for model improvement. Secondly, according to most of model input can be delivered via text file, input data error has to be corrected only via text file that is quite time consuming. Therefore model input needs to be improved to be able to interact with users via window facility. Thirdly, model output consisting of stage and discharge hydrographs are also delivered via text file, the output data in graphic form should be created. Finally, model output should be able to be transmitted to demonstrate on the Geographic Information System that will then be used to create flood risk mapping and other facilities that would be very useful for decision making.

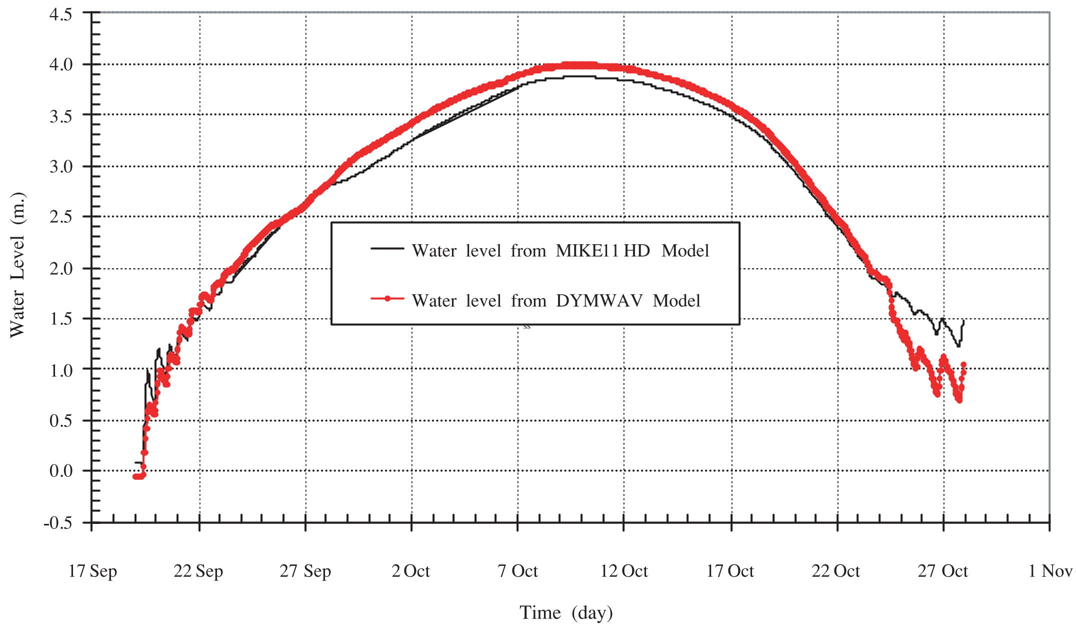

Figure 3 Stage hydrograph comparison between the DYMWAV model and the MIKE 11 HD model at Kgt.1 station.

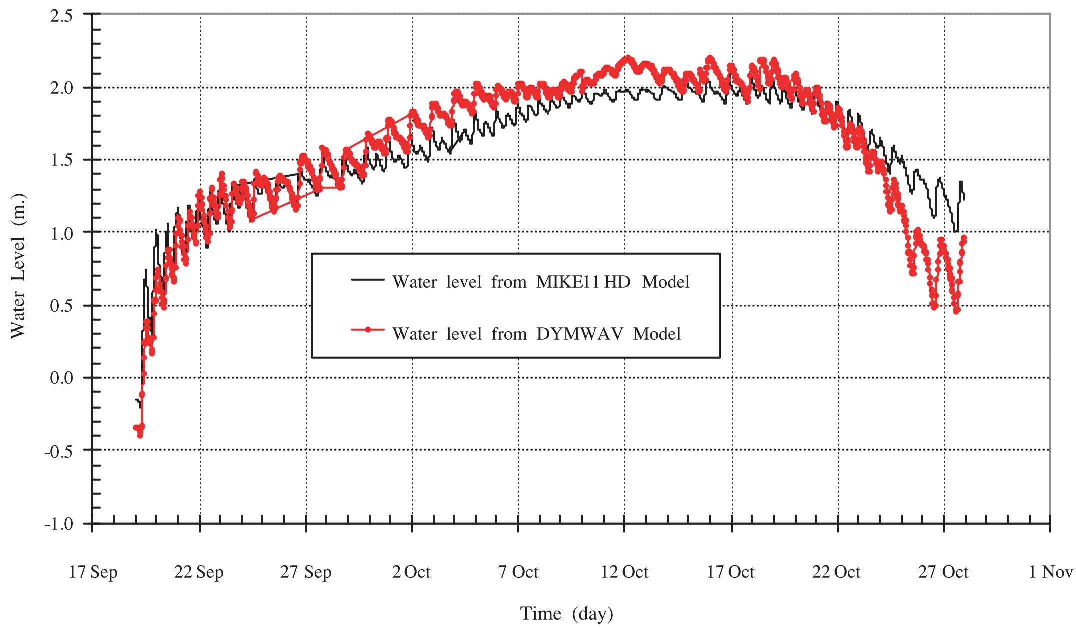

Figure 4 Stage hydrograph comparison between the DYMWAV model and the MIKE 11 HD model at Bang Kanak Regulator.

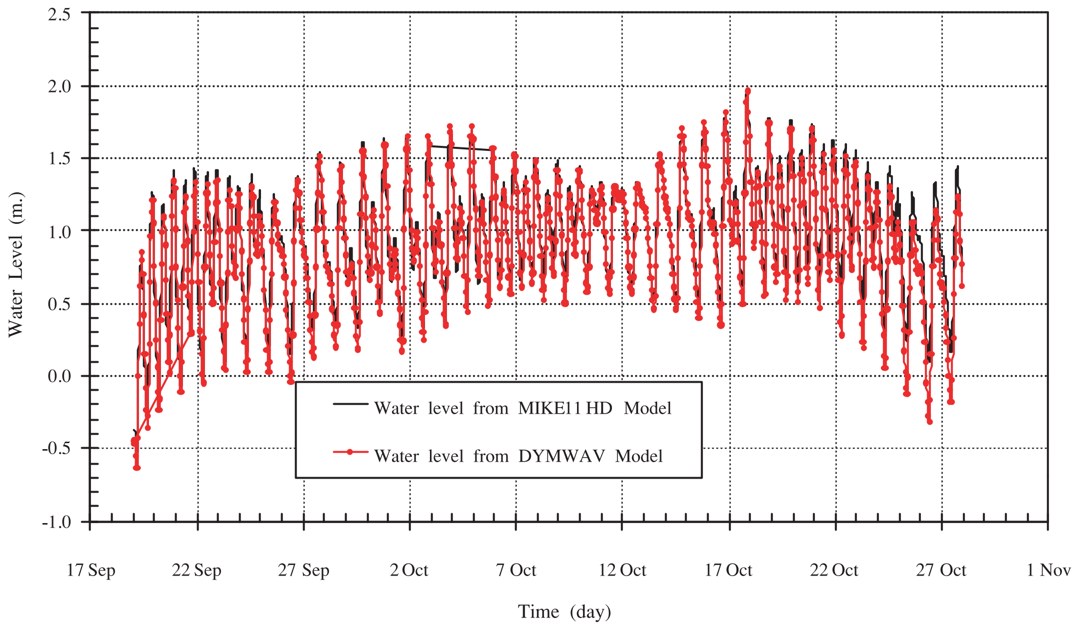

Figure 5 Stage hydrograph comparison between the DYMWAV model and the MIKE 11 HD model at Tah Khai Regulator.

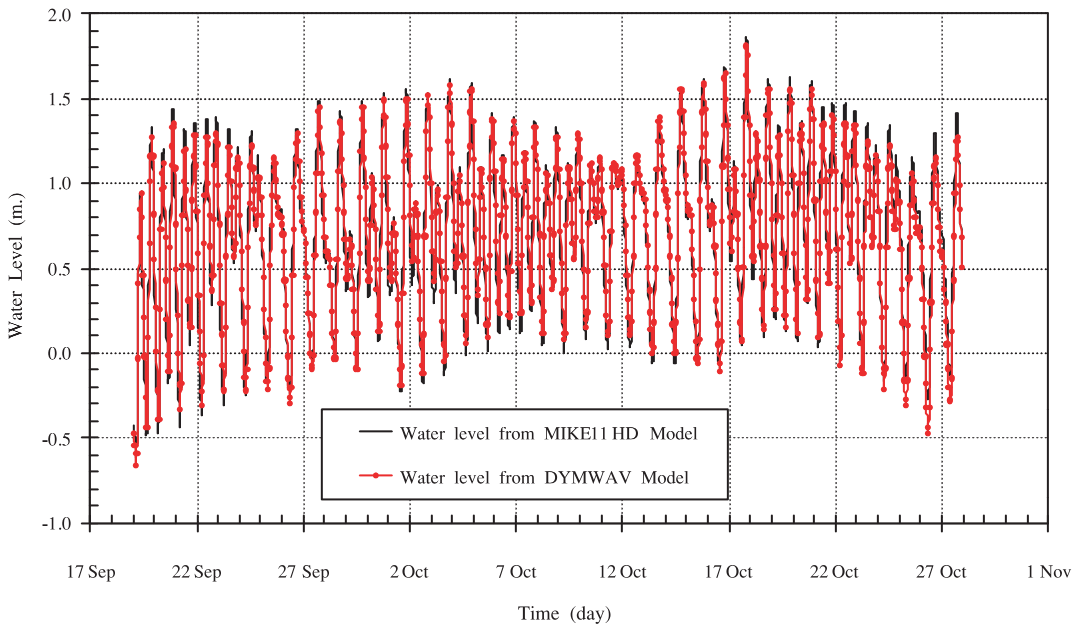

Figure 6 Stage hydrograph comparison between the DYMWAV Model and the MIKE 11 HD model at Tah Tour Regulator.

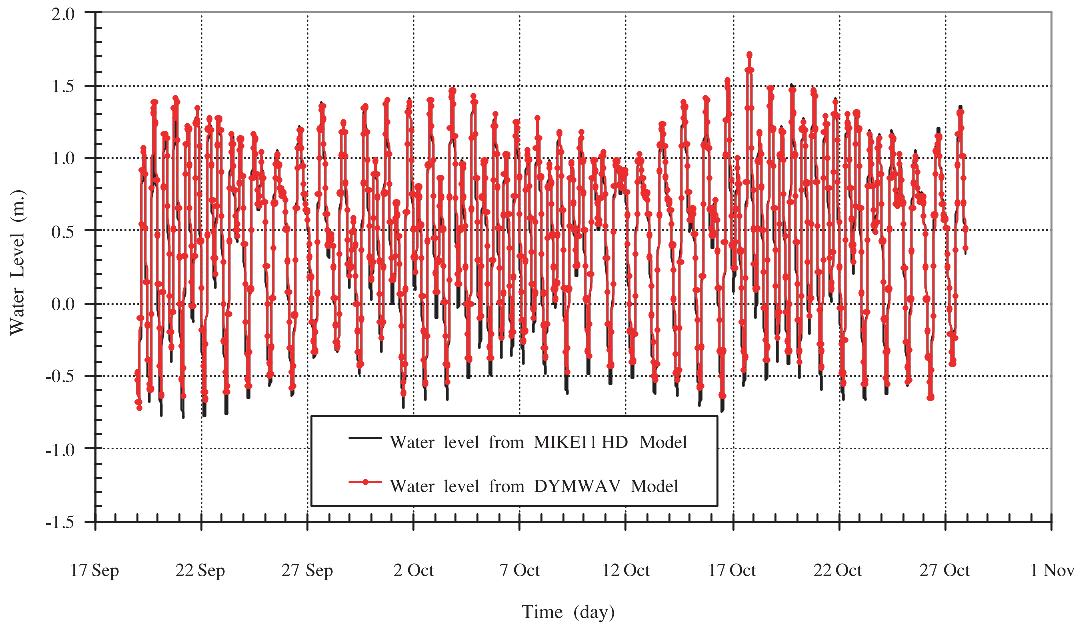

Figure 7 Stage hydrograph comparison between the DYMWAV Model and the MIKE 11 HD model at Paktaklong Regulator.

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